Greek Letters

α = defined by Equation (9)
ρ = density, lb.-mole/cu.ft.
ψ = adjoint variable in maximum principle formulation

Subscripts

A = diethyl adipate
B = hydroxyl ion
R_1 = monoethyl adipate ion
R_2 = adipate ion
S = shell side
T = tube side
i = inside
in = inlet
o = outside
1, 2 = first and second saponification reactions

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Hydrodynamic Measurements for Imperfect Mixing Processes: Newtonian Fluids

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The response of a continuous flow mixing system to a step change in an input variable is discussed from the standpoint of dimensional analysis. This idea is presented as an alternative to using zone models for predicting residence time distributions in systems where imperfect mixing occurs.

Mixing experiments were performed in a cylindrical flat bottomed tank geometrically similar to tanks commonly used for industrial processes. The response of the system to a step change in feed concentration was observed. Initially the tank contained a salt solution. At the start of an experiment, a stream of salt-free diluent was introduced at the top of the tank and a stream of the salt solution was drained from the bottom, keeping the liquid volume in the tank constant. The salt concentration in the output stream was measured continuously after the start of the experiment.

The experimental results are correlated in terms of dimensionless variables, and the variables affecting the mixing process most strongly are determined. The results show that variations in throughput rate or in impeller shape or rotational speed affect the mixing process much more than does impeller position. The data are also compared with models proposed by other authors, and the model constants are evaluated.

Recently, experimenters have been studying stirred-tank systems from the fundamental point of view, using the principles of fluid mechanics, energy transport, and mass transport to predict the behavior of the system. Several authors (*1, 2*) have reported detailed velocity profiles within the mixing tank itself and have related the profiles to the impeller speed, fluid properties, and geometric variables. Logical extensions of such work would be to determine temperature and concentration distributions in the stirred tank and correlate the data in a similar manner. Other workers (*3* to *13*) have analyzed the stirred-tank system from a macroscopic viewpoint. Power requirements have been related to geometric and process variables using this approach, and energy and mass transfer processes have been studied. Many of these studies have been carried out using a continuous flow stirred tank. A schematic flow

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diagram of this type of system used for the present work is shown in Figure 1. The characteristics of the input and output streams are measured and related to the geometric and process variables which describe the operating conditions of the system. A simple experiment is to determine the response of the system to a step change in the input concentration by measuring the concentration in the outlet stream as a function of time. $c_f = c_i$ before the step change occurs, and $c_f = 0$ after the step change has occurred. The macroscopic balance for solute is

$$\frac{d}{dt}(\bar{c}) = \frac{F}{V} c_o \tag{1}$$

This equation is to be integrated using the initial condition

$$\text{At } t = 0,\ c_o = c_i \text{ and } \bar{c} = c_i \tag{2}$$

If perfect mixing takes place in the tank

$$\bar{c} = c_o \tag{3}$$

When this condition is not satisfied, imperfect mixing is occurring in the tank. Equation (1) is solved together with Equations (2) and (3) to give

$$\frac{c_o}{c_i} = e^{-\frac{F}{V}t} = e^{-Dt} \tag{4}$$

Equation (4) is the basis for most of the work dealing with imperfect mixing, since it provides a reference response to which the actual system response can be compared. In describing stirred-tank behavior, most authors have sought to define a model containing parameters which are easily determined for any particular system and which characterize unequivocally the deviation from perfect mixing behavior.

Danckwerts (*6*) introduced the idea of residence time distribution for a general flow process. This has been the starting point for a great deal of other work and has been reviewed by Levenspiel (*8, 14*) and by Bischoff and McCracken (*15*). The fundamental data needed for applying Danckwerts' analysis to a stirred-tank system is the so-called *F* diagram illustrated in Figure 2 for a step decrease in normalized feed stream concentration. Danckwerts defines a measure of the deviation from perfect mixing, the segregation, as half the area enclosed between the *F* diagram for perfect mixing and the observed *F* diagram. He also discusses predicting *F* diagrams for large equipment using model data. Václavek (*11*) has discussed mixing in approximately the same way as Danckwerts, but uses the idea of transfer functions to relate the input and output streams. The segregation or the transfer function for any system must be determined experimentally from the *F* diagram. Several authors have sought general methods for predicting the *F* diagram for a system in which imperfect mixing takes place. Most of these authors divide the system into various zones, each having different characteristics.

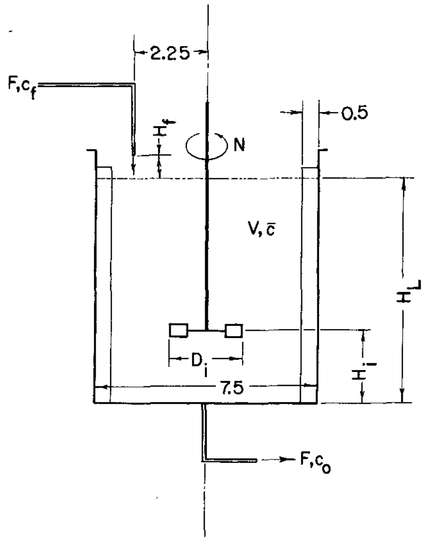

Fig. 1. Flow diagram and dimensions for the mixing system. The dimensions are given in inches. The feed line is located 2.25 in. from the centerline at an angle of 38 deg. to one baffle. The dimensions not specified in the figure are as folllows: $D_i = 2.5$, 3.0, and 4.0 in. $H_i = 2.6$, 3.6, and 4.2 in. $H_L = 7.6$ and 9.0 in. $H_f = 1.0$, 1.2, and 1.5 in.

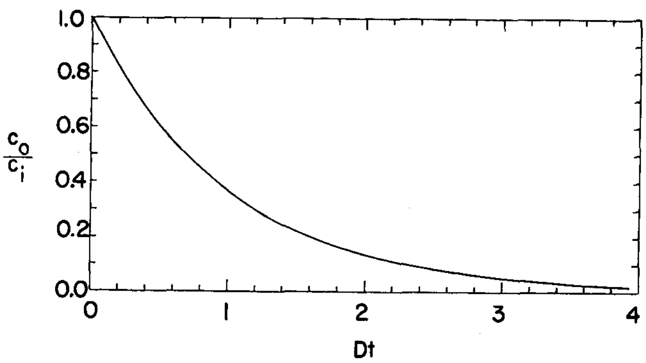

Fig. 2. Danckwerts' *F* diagram for perfect mixing

Levenspiel (*8*) postulates four types of zones: plug flow, perfect mixing, stagnant fluid, and plug flow plus diffusion. The fluid flowing through the process vessel can pass from one zone to another, or it can bypass a particular zone, or it can be recycled within a single zone. In his textbook, Levenspiel (*14*) discusses the behavior of stirred tanks.

Cholette and Cloutier (*3, 4, 7*) divide the vessel into perfect mixing, piston flow, and stagnant zones. Equations describing the response of the system to a step change in feed are developed for flow from one zone to another. They relate the models to an effective volume for mixing and to three constants all of which must be evaluated from experimental data. Experimental results are reported for tanks 30 and 60 in. in diameter agitated by impellers 2.5, 3, and 4 in. in diameter. This tank geometry does not correspond to the geometry commonly used in industrial mixing applications, but the geometry selected does produce imperfect mixing over a wide range of impeller speeds. The model constants and effective volume are determined as functions of impeller size and shape, impeller speed, feed position, feed rate, and baffle size.

Manning and co-workers (*12, 13*) have formulated a model for imperfect mixing in a stirred-tank chemical reactor. The model is presented specifically for mixing with a second-order reaction taking place. The model should also be applicable for a system in which no reaction is occurring. The basic feature of the model involves relating the outlet reactant concentration to the ratio of the impeller pumping capacity and the throughput rate. Data are presented to show the effect of feed position on the outlet concentration. Berresford et al. (*16*) also recommend the ratio Q/F as the best scale-up criterion to use in continuous flow systems.

Taguchi et al. (*10*), instead of dividing the vessel into zones, divide the throughput for a continuous flow mixing system into two fractions: a piston flow fraction ϕ and a perfect mixing fraction $1 - \phi$. Experimental results for 100 liters of solution being mixed in a fermentation tank 500 mm. in diameter are reported, and ϕ is evaluated from the data. A step change was produced by adding dye to the feed solution. The experiment was performed using water and aqueous carboxymethyl cellulose solutions. ϕ is reported as a function of impeller speed and dilution rate D for turbine and marine propellers.

Corrigan and Beavers (*5*) present results illustrating the effect of introducing a stagnant area into the mixer. A definitely measurable stagnant zone was produced by placing a sponge in the tank. The fluid outside the stagnant zone is assumed to be perfectly mixed. The model also includes the effect of channeling. Not much data are presented to verify the model.

The objective in formulating zone models is to be able to predict the F diagram introduced by Danckwerts without having to make experimental measurements for a particular system. The constants in the zone models must be evaluated from experimental data, and these constants will depend on geometric and processing variables. Unless a general correlation for these constants is obtained, there is no clear advantage in using zone models over reporting actual F diagrams.

In the present work, some general aspects of the manner in which F diagrams and zone model constants depend on the geometric and processing variables are discussed. To be most useful, experimental data illustrating the effects of different variables should be presented in dimensionless form. If this is done, scale-up criteria can be selected easily. Experimental data are presented to illustrate the importance of some variables on the operation of a continuous flow mixing system of a geometry similar to that commonly used for industrial applications.

DIMENSIONAL ANALYSIS

If the results of experimental measurements for continuous flow stirred tanks are to be presented in dimensionless form, the most physically significant dimensionless groups should be used for formulating a general data correlation. This can be done using the techniques of dimensional analysis. Stewart (*17*) reviews the features of the two methods commonly used—analysis of the governing equations and the Buckingham analysis.

When the analysis of the governing equations is used, the proper variables are selected automatically, and the dimensionless groups physically most pertinent for the system appear as a matter of course. The set of dimensionless groups obtained may not be a complete set. Significant physical variables may be omitted from the governing equations in two ways. First, assumptions must be made when the differential equations are derived, and these assumptions limit the applicability of the equations. Second, it is often difficult to specify boundary conditions well enough to include all the important variables which influence the transport processes in a particular system.

An exact physical understanding of the transport processes occurring in the system is not necessary if the Buckingham analysis is used. Once the important variables are selected, complete sets of dimensionless groups can be determined using the methods described by Langhaar (*18*). The Buckingham method includes no systematic method for selecting variables which are important in any specific system.

For analyzing the continuous flow stirred-tank system, a combination of both dimensional analysis methods can be used. Since the analysis of the governing equations gives the dimensionless groups physically most meaningful for the system, that complete set of dimensionless groups obtained by the Buckingham method should be selected which contains the same dimensionless groups as those which arise from the analysis of the governing equations. This will give the complete set of dimensionless groups most meaningful physically for the system.

Bird, Stewart, and Lightfoot (*19*) give the dimensional analysis for the governing equations describing the conservation of total mass, linear momentum, and mass of solute species. The equations are restricted to systems in which there is isothermal, incompressible flow of a fluid having constant physical properties and in which only pressure and gravity act on the fluid. The results of the dimensional analysis of the differential equations are

$$\mathbf{v}^* = v^*(x_j^*, t^*, N_{Re}, N_{Fr}) \tag{5}$$

$$p^* = p^*(x_j^*, t^*, N_{Re}, N_{Fr}) \tag{6}$$

$$c^* = c^*(x_j^*, t^*, N_{Re}, N_{Sc}, \mathbf{v}^*) \tag{7}$$

The characteristic quantities chosen are given in Table 1*a*. The dimensionless variables obtained using these characteristic quantities are given in Table 1*b*. For the continuous flow stirred-tank system, the response of the system is going to be determined by measuring the outlet concentration. If a baffled tank is used, the liquid surface remains level, and gravitational effects will be negligible, allowing the Froude number to be eliminated. If only one solute species is considered, N_{Sc} will be constant. Under these conditions, the dimensionless outlet concentration can be represented by a combination of Equations (5), (6), and (7):

$$c_o^* = c_o^*(t^*, N_{Re}) \tag{8}$$

Equation (8) gives an incomplete set of dimensionless groups, since any dimensionless groups which arise from the boundary conditions have not been included. If the dimensionless outlet concentration is to be found, a complete set of boundary conditions in dimensionless form is also required. The dimensionless velocity and concentration must be specified at all points on the boundary. This can be done easily when the feed rate is zero by using the assumption of no slip at solid-fluid interfaces. This introduces a set of ratios relating all dimensions of the system to the impeller diameter, and these ratios then appear in Equation (8). For a nonzero feed rate, the velocity distribution at the entrance and exit points is distorted in a way which is difficult to determine but which is related to the feed rate. This means that a dimensionless group involving the feed rate will also appear in the dimensionless boundary conditions. It is difficult to determine the form of such a di-

TABLE 1*a*. CHARACTERISTIC QUANTITIES USED FOR THE DIMENSIONAL ANALYSIS OF THE MIXING SYSTEM

Characteristic length = D_i
Characteristic velocity = ND_i
Characteristic concentration difference = $(c_i - c_f)$

TABLE 1*b*. DIMENSIONLESS VARIABLES USED IN DESCRIBING THE MIXING SYSTEM

Velocity = $\mathbf{v}^* = \dfrac{\mathbf{v}}{ND_i}$
Time = $t^* = tN$
Pressure = $p^* = \dfrac{p - p_0}{\rho N^2 D_i^2}$
Solute concentration = $c^* = \dfrac{c - c_f}{c_i - c_f}$
Position coordinate = $x_j^* = \dfrac{x_j}{D_i}$

mensionless group, and this limits the applicability of the method of analyzing the governing equations.

In order to apply the Buckingham analysis to the continuous flow stirred-tank system, a list of physically significant variables must be selected. Any variables can be included, and this makes it easy to include the feed rate.

For an isothermal system, length, time, and mass are chosen as the fundamental dimensions. The variables chosen are t, N, D_i, C_i, μ, ρ, D_T, H_L, H_i, and $D = F/V$. Then, the outlet concentration can be described as a function of the following form if the feed concentration is zero:

$$c_o = c_o(c_i, t, D, N, D_i, \mu, \rho, D_T, H_L, H_i, L_j) \tag{9}$$

One complete set of dimensionless groups which can be formed from this list of variables is

$$c^* = c^*\left[(tN), \left(\frac{D_i^2 N\rho}{\mu}\right), \left(\frac{D_i^2 D\rho}{\mu}\right), \left(\frac{D_T}{D_i}\right), \left(\frac{H_L}{D_i}\right), \left(\frac{H_i}{D_i}\right), \left(\frac{L_j}{D_i}\right)\right] \tag{10}$$

This is the set of greatest interest, because it contains many of the same dimensionless groups which appeared in the analysis of the governing equations. These are the groups most closely related to the physical behavior of the system. This means that experimental data for a system of a given geometric configuration can be represented as c_o/c_i versus tN with N_{Re} as a parameter if $N_D = (D_i^2 D\rho/\mu)$ is kept constant.

The results of this dimensional analysis operation are not completely consistent with the methods used by other authors for reporting their data. The philosophy of the dimensional analysis method causes the apparent discrepancy. The first part of the philosophy is to look for a set of dimensionless groups to be used for describing all types of operations which occur in stirred tanks: power consumption, mass transfer, transient response, etc. The second part of the philosophy as stated above is that the results from analyzing the governing equations and from applying the Buckingham analysis should be the same as far as possible.

Consider the problem of correlating power consumption data for stirred tanks. The analysis of the governing equations as described by Bird, Stewart, and Lightfoot (20) gives the commonly used method for correlating such data.

$$N_{Pow} = N_{Pow}\left[(tN), \left(\frac{D_i^2 N\rho}{\mu}\right), \left(\frac{D_i N^2}{g}\right)\right] \tag{11}$$

The form of the Reynolds number which appears here is the thing of most interest. To obtain $N_{Re} = D_i^2 N\rho/\mu$ in the dimensionless governing equations, ND_i must be chosen as the characteristic velocity for the system. This choice leads automatically to tN as the dimensionless time to be used in the dimensionless governing equations. Since momentum transport is always involved in liquid mixing operations, it will always be necessary to consider the dimensionless equation governing momentum transport in analyzing any mixing system. This means that the dimensionless time tN should be used in all applications.

This is the idea which is inconsistent with some of the results reported by other workers. The results for the perfect mixing case could also be used as a guide for selecting the form for the dimensionless time. Equation (4) is already in dimensionless form and predicts the behavior of the system if perfect mixing takes place. For this case the Reynolds number does not appear, and the dimensionless concentration is a unique function of tD, a form of dimensionless time different from the one deduced previously to be the most physically significant. If, instead, the response for a system in which perfect mixing takes place is presented in terms of tN, an additional term D/N appears, and the dimensionless concentration is no longer a unique function of the dimensionless time. This seems to indicate that using tN gives results which are not reduced to the most general form. This problem is not important for tanks in which imperfect mixing is taking place, because the response curve depends both on the impeller speed and on the throughput rate. No advantage with respect to one form of the dimensionless time or the other is evident on the basis of the dimensional analysis only. One or the other form may be more useful in a particular model.

EXPERIMENTAL TECHNIQUES

Experiments were carried out in a cylindrical flat-bottomed tank made from polymethylmethacrylate tubing as shown in Figure 1. Four baffles were spaced equally around the tank wall to eliminate vortex formation.

A step change in the solute concentration in the input stream was produced by the following method. The tank is filled with an aqueous solution of about 1.5 molar sodium chloride, and at a certain instant, the flow is started—pure water is directed into the tank at the top and the salt solution flows out from the bottom. The chloride ion concentration is measured continuously in the outlet stream as the solution passes through a measurement cell containing a small silver-silver chloride electrode. The potential of this small working electrode measured with respect to a large silver-silver chloride reference electrode immersed in a solution of constant chloride ion concentration is a function of the chloride ion concentration in the flowing stream. This potential difference is recorded as a function of time and gives the basic data for analyzing the system response. Complete details about the construction and operation of the equipment are available elsewhere (9).

A step change in the inlet solute concentration is produced by starting with more salt solution in the tank than is wanted for the experiment. The solution is allowed to drain from the tank at a preadjusted rate. At the instant the level of the solution in the tank reaches the desired point, the stop valve in the inlet line from the constant head tank is opened and the recorder chart motor is started. The flow rate from the constant head tank has previously been adjusted to the same flow rate as that in the drain line. An error is introduced by this method of operation. Before the step change occurs, no liquid is flowing into the tank at the top. Introducing a stream at the top of the tank will have some effect on the flow patterns in the tank. At the start of an experiment, before the step change, the level is changing a little also. This will also have an effect on the flow patterns. In the present work, it has been assumed that these initial disturbances decay rapidly and a steady state flow pattern is quickly established.

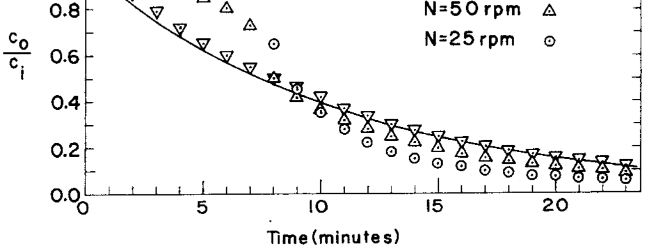

Fig. 3. Typical data for a series of mixing experiments. $D_i = 2.5$ in. (paddle impeller), $H_i/D_i = 1.68$, $H_L/D_i = 3.60$, $D_T/D_i = 3.0$, $W/D_i = 0.2$, $D = 0.092$ min.$^{-1}$. The solid line represents Equation (4).

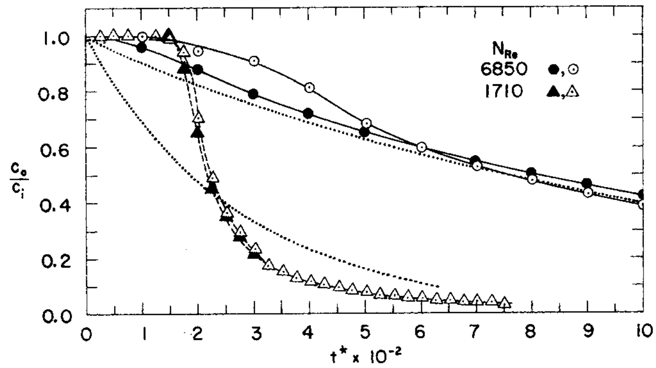

Fig. 4. Effect of Reynolds number on imperfect mixing for a paddle impeller (solid points) and a marine propeller (open points). For this case, $D_i = 2.5$ in. and $D_T/D_i = 3.0$, $H_L/D_i = 3.6$, $H_i/D_i = 1.7$, $W/D_i = 0.20$, $N_D = 6.3$. The dotted lines represent Equation (4).

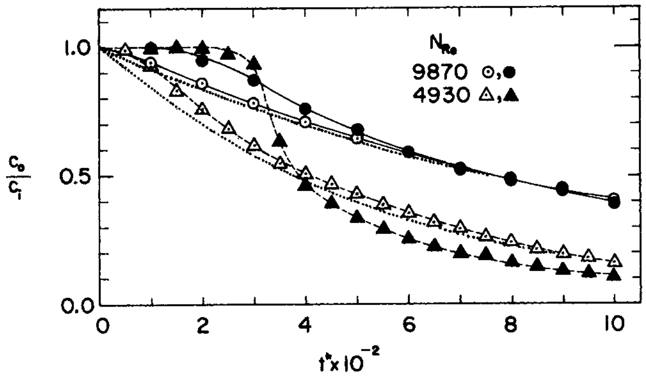

Fig. 5. Effect of Reynolds number on imperfect mixing for a paddle impeller (open points) and a marine propeller (solid points). For this case, $D_i = 3.0$ in. and $D_T/D_i = 2.5$, $H_L/D_i = 3.0$, $H_i/D_i = 1.4$, $W/D_i = 0.17$, $N_D = 9.0$. The dotted lines represent Equation (4).

Another error may arise because of the density changes which occur in the tank during the course of an experiment. The density of 1.5 molar sodium chloride solution is 5% greater than the density of pure water. When the pure water feed is introduced at the top of the tank, a stratification could occur during the first part of an experiment. Any error due to density differences has been neglected in the work reported here.

EXPERIMENTAL RESULTS

Typical raw data are shown in Figure 3, where c_o/c_i is plotted as a function of time for several impeller speeds. Enough data are not available to enable determining completely the effects of the many dimensionless geometric and process variables which appear in the dimensional analysis. Some basic trends can be deduced using the data plotted in dimensionless form. The variable which evidently influences the mixing process most is the Reynolds number. Figure 4 shows this effect both for paddle impeller and marine propeller impellers. The curve representing perfect mixing is also shown. As the Reynolds number is increased, the response curve becomes more nearly coincident with the response curve for perfect mixing [Equation (4)]. These curves also can be used to show the effect of impeller shape. At lower Reynolds numbers ($N_{Re} = 1{,}710$ in Figure 4), the impeller shape does not seem to have a great effect: the curves are nearly coincident. At higher Reynolds numbers ($N_{Re} = 6{,}850$ in Figure 4), the impeller shape becomes more important, since the dilution curves diverge more markedly from each other. This is a general trend as is illustrated in Figures 5 and 6 for different impeller diameters. In each case the curve for perfect mixing is also shown. For the particular set of dimensionless groups chosen, the perfect mixing curve depends on the Reynolds number, because tN rather than tD was chosen as the dimensionless time. The data show that as the Reynolds number increases, the curves become more nearly coincident to the curve for perfect mixing. The critical value for N_{Re} to produce perfect mixing in a continuous flow system has not yet been determined.

The dimensionless dilution curves (c_o/c_i versus t^*) can be used to deduce something about the effects of minor geometric variations. Figure 7 shows the effect of changing the distance between the impeller and the tank bottom. Only a small effect is evident in this case. The effect of other geometric variations could be deduced in the same way.

The ultimate goal of this work is to present a set of generalized charts for various mixer geometries from which dilution curves for any particular mixing system can be predicted. A convenient form for such charts seems to be as shown in Figure 8, where c_o/c_i is plotted as a function of N_{Re} with t^* as a parameter. Such a chart applies only for a specific system geometry and for a constant value of N_D. When plotted this way, c_o/c_i seems to be a simple function of N_{Re}. Since geometric similarity is not maintained exactly for these data, some scatter is evident. Further work is in progress to determine complete

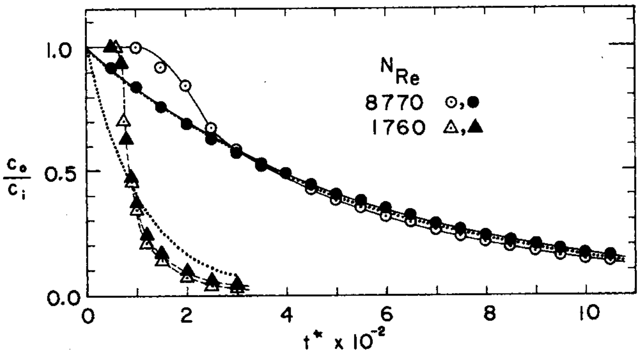

Fig. 6. Effect of Reynolds number on imperfect mixing for a paddle impeller (solid points) and a marine propeller (open points). For this case, $D_i = 4.0$ in. and $D_T/D_i = 1.9$, $H_L/D_i = 2.25$, $H_i/D_i = 1.05$, $W/D_i = 0.13$, $N_D = 15.9$. The dotted lines represent Equation (4).

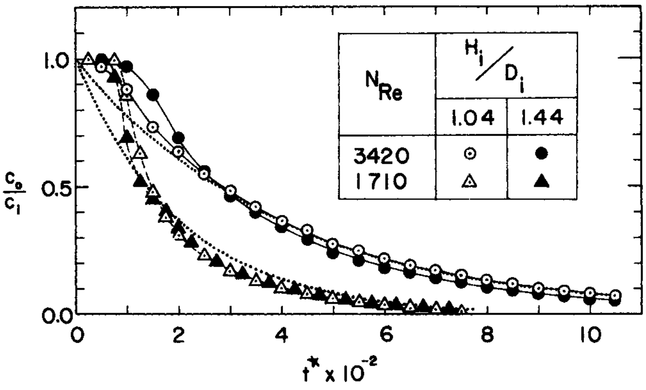

Fig. 7. Effect of impeller position on imperfect mixing. For this case, $D_i = 2.5$ in. (six-blade paddle impeller), and $D_T/D_i = 3.0$, $H_LD/_i = 3.04$, $H_i/D_i = 1.04$, $W/D_i = 0.20$, $N_D = 8.6$. The dotted lines represent Equation (4).

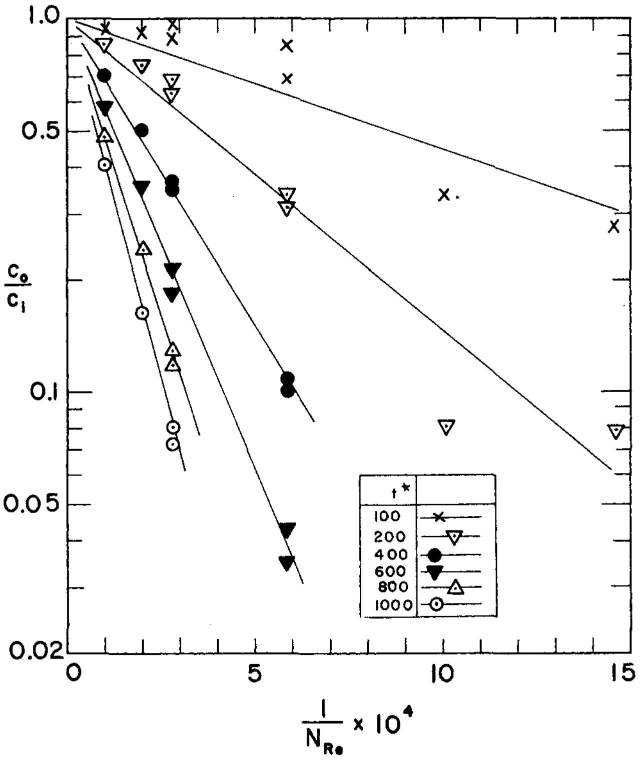

Fig. 8. Generalized plot for imperfect mixing data. For this case, $N_D = 8.6$ to 9.1, $D_T/D_i = 2.5$ to 3.0, $H_L/D_i = 3.0$, $H_i/D_i = 1.0$ to 1.4, $W/D_i = 0.17$ to 0.20.

generalized data for mixing systems of a variety of geometric configurations.

Once the generalized correlations are available, they can be used for scale-up calculations, for predicting the effect of changes in geometry or flow conditions for a particular system, and for mixer design. For example, Danckwert's *F* diagram can be obtained from a set of such generalized curves merely by reading the values of c_o/c_i at various values of t^* and at constant N_{Re}. These values are then used to plot c_o/c_i versus t^* $(D/N) = tD$.

COMPARING THE DATA TO OTHER MODELS

The data also can be analyzed using the methods proposed by other authors. Segregation, as defined by Danckwerts (*6*), can be calculated from the *F* diagram. Since the segregation depends on the shape of the response curve, the type of dimensional analysis described above can be applied. The segregation could then be correlated as a function of Reynolds number, dilution number, and the geometric variables. The segregation gives a qualitative idea about the deviation from perfect mixing and could be used to determine the Reynolds number at which perfect mixing could be expected to begin in a specific system. The segregation is not the most useful quantity to use for describing the response of the continuous flow mixing system, because the *F* diagram cannot be generated simply by knowing the segregation.

Cholette and co-workers (*3*, *4*, *7*) propose a model for representing the response of a continuous flow mixing system by an exponential function of the form

$$\frac{c_o}{c_i} = ne^{-\frac{n}{m}Dt} \tag{12}$$

m is called the level of mixing, and $(1 - n)$ is the fraction of fluid short-circuited. The appearance of response data plotted according to Equation (12) is shown in Figure 9. The equation represents the data best at higher Reynolds numbers. At lower Reynolds numbers, significant deviations occur for $Dt < 1.0$. Since *m* and *n* depend on the shape of the response curve, the dimensional analysis method discussed previously can be applied. The parameters *m* and *n* could be correlated as functions of Reynolds number, dilution number, and system geometry. The effect of Reynolds number upon *m* and *n* is shown in Figure 10 for typical data from the continuous flow system studied. Only data for $Dt > 1.0$ were used for these calculations. There does seem to be a general trend for the data: each parameter decreases as the Reynolds number decreases. Insufficient data are available to enable a general correlation to be presented. Even if correlations for *m* and *n* were available, the *F* diagram could not be reproduced accurately because of the deviations for $Dt < 1.0$. Patel (*9*) presents a more complicated model similar to Equation (12) but containing four constants instead of two. This model represents the data accurately, but would not be as useful for design purposes as a simpler model.

Manning and co-workers (*12*, *13*) present another type of model for a continuous flow mixing system. Aggarwal and Manning (*21*) show how the model can be applied to Patel's data. There is good agreement except at low Reynolds numbers. There is a distinct advantage over some of the other models proposed. In order to apply the model, only impeller discharge rates are needed. Wolf and Manning (*2*) and Cooper and Wolf (*22*) discuss impeller discharge rates with respect to impeller geometry, but no mention is made of the effect of tank geometry.

CONCLUSIONS

The experimental data obtained show that imperfect mixing does occur in a stirred-tank system having a geom-

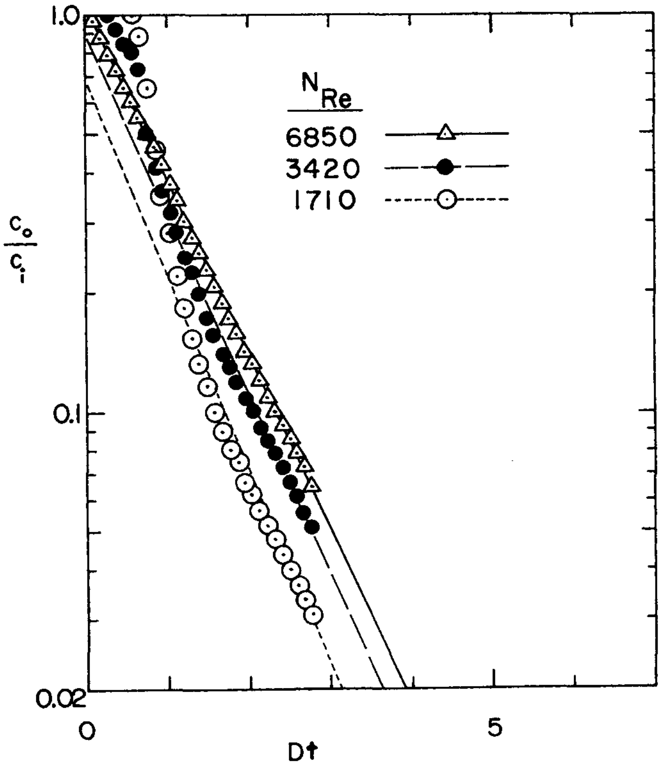

Fig. 9. Imperfect mixing data plotted according to Equation (12). $D_i = 2.5$ in. (paddle impeller), $H_i/D_i = 1.68$, $H_L/D_i = 3.60$, $D_T/D_i = 3.0$, $W/D_i = 0.20$, $D = 0.092$ min.$^{-1}$.

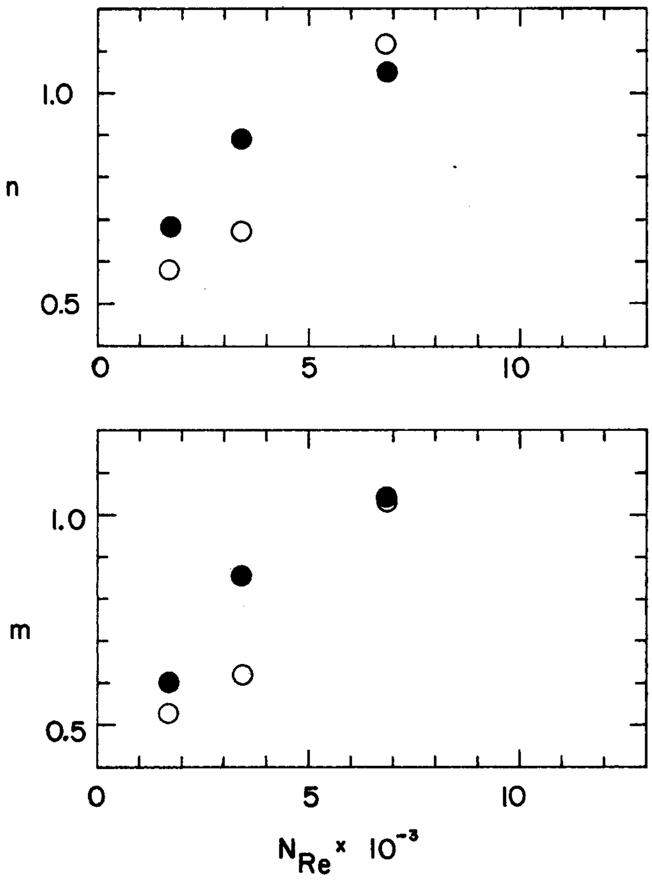

Fig. 10. Effect of Reynolds number on level of mixing *m* and fraction of fluid not short-circuited *n* for a paddle impeller (solid points) and a marine propeller (open points). $D_i = 2.5$ in. $D_T/D_i = 3.0$, $H_i/D_i = 1.68$, $H_L/D_i = 3.60$, $W/D_i = 0.20$, $D = 0.0092$ min.$^{-1}$.

etry similar to that used in industrial applications. Dimensional analysis gives a systematic method of treating the effect of geometric and process variables, but enough data are not available to give a complete discussion of all effects. Simplified models give an approximate representation of the response curves, but for low Reynolds numbers or for the initial part of the process, such models deviate considerably from the true behavior of the system. The best model to use for design purposes appears to be the one proposed by Manning and co-workers.

NOTATION

c = molar concentration of a particular solute species
$\bar{c}$ = average molar concentration of solute within the mixing tank
c_i = initial molar concentration of solute in the mixing tank
c_f = molar concentration of solute in the feed stream
c_o = molar concentration of solute in the outlet stream
D_i = impeller diameter
D_T = tank diameter
$D = \dfrac{F}{V}$ = dilution rate
$\mathfrak{D}_m$ = effective binary diffusion coefficient for solute
F = volumetric flow rate of the feed stream
$\mathbf{g}$ = gravitational acceleration vector
g = magnitude of gravitational acceleration
H_i = impeller position, distance from tank bottom
H_L = liquid level, distance from tank bottom to liquid surface
H_f = distance from liquid surface to feed point
L_j = j^{th} dimension in the mixing system
N = impeller rotational speed
$N_D = \dfrac{D_i^2 D\rho}{\mu}$ = dilution number
$N_{Fr} = \dfrac{N^2 D_i}{g}$ = Froude number
$N_{Re} = \dfrac{D_i^2 N\rho}{\mu}$ = Reynolds number
$N_{Pow} = \dfrac{P}{\rho N^3 D_i^5}$ = Power number
$N_{Sc} = \dfrac{\mu}{\rho \mathfrak{D}_m}$ = Schmidt number
P = power consumption
p = pressure
p_o = reference pressure
Q = impeller discharge rate
t = time
$\mathbf{v}$ = velocity vector
v_j = j^{th} component of the velocity vector
V = volume of the mixing tank
x_j = spatial coordinate in j^{th} coordinate direction

Greek Letters

ϕ = fraction of plug flow
μ = fluid viscosity
ρ = fluid density

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